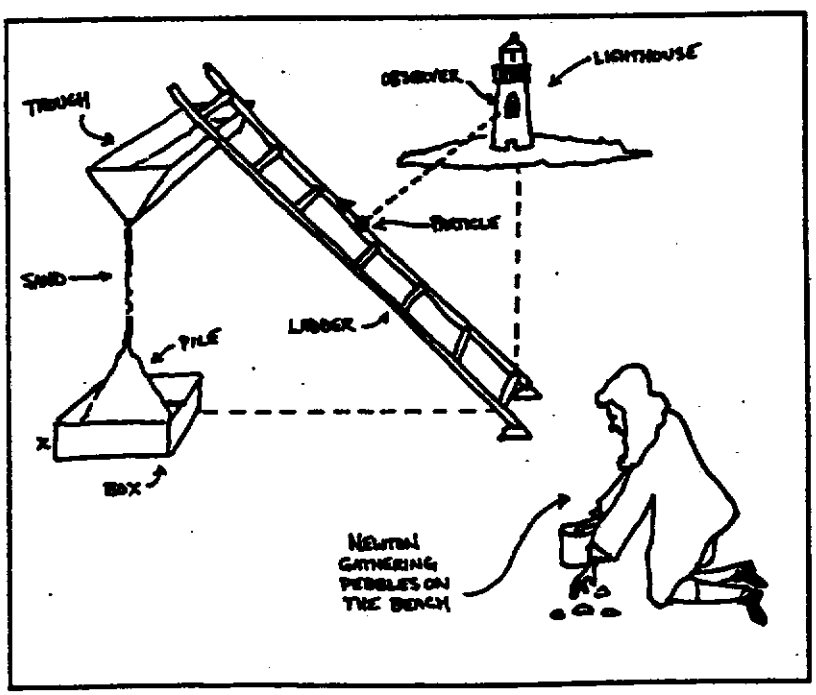
DAN KENNEDY

The All-Purpose Calculus Problem

Here's a calculus problem to end all calculus problems. (And you thought your professor assigned you hard ones!) See how many familiar themes you can find embedded in this problem.

A particle starts at rest and moves with velocity $v(t) = \int_1^t e^{-x^2} dx$ along a 10-foot ladder, which leans against a trough with a triangular cross-section two feet wide and one foot high. Sand is flowing out of the trough at a constant rate of two cubic feet per hour, forming a conical pile in the middle of a sandbox which has been formed by cutting a square of side x from each corner of an 8" by 15" piece of cardboard and folding up the sides. An observer watches the particle from a lighthouse one mile off shore, peering through a window shaped like a rectangle surmounted by a semicircle.

(a) How fast is the tip of the shadow moving?
(b) Find the volume of the solid generated when the trough is rotated about the y-axis.
(c) Justify your answer.
(d) Using the information found in parts (a), (b), and (c) sketch the curve on a pair of coordinate axes.



DAN KENNEDY is chair of the mathematics department at the Baylor School, Chattanooga, TN and is chair of the AP Calculus Committee.